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Natural convection in an annulus between coaxial horizontal cylinders with internal heat generation

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Abstract

Natural convection of gas ($Pr = 0.7$) between two horizontal coaxial cylinders with uniform internal heat generation is numerically investigated. Such a problem plays an important role in the analysis of lasers in which a similar circuit of a heat-conducting path is used. It has been found that the behavior of the system critically depends on three parameters including the inverse relative gap width σ(=diameter of the inner cylinder/gap width), the Rayleigh number Ra and the modified Rayleigh number Ra_T which describes heat generation. In the case $Ra_T = 0$ our results coincide with the known ones. It has been established that in such a system there exist two types of fluid flow for low Rayleigh numbers with different vortex structure. Optimization of the corresponding coaxial laser system has been analyzed.

1. Introduction

The main problem associated with laser and discharge stability is the increase of translation temperature with the power increase. The increase in temperature leads to acceleration of relaxation processes, nonequilibrium decrease, contraction and breakdown of generation. The convective flow systems are frequently used to achieve a high power output, but they are bulky and difficult in operation. The search of a new scheme of the organization of the discharge is, therefore, constantly under way. In recent years, coaxial waveguides have been extensively used [1,2]. Advantages of such system are obvious. The axis of the cylinder, where the temperature is maximal, can be effectively cooled. Certainly, there are additional problems related to radiation characteristics of such a laser, but these questions will not be considered in the present paper. The primary attention will be focused upon a problem of the heat transfer. In fact, convection in a system of coaxial cylinders at fixed temperatures of cylinders leads to approximately double increase of a flow, compared with the flow determined only by the heat conductivity [3–5].

The purpose of the present work is to investigate a stationary convective flow in the nonequilibrium annulus between two coaxial cylinders. The simplest model of uniform internal heat generation is considered which enables one to simulate the real energy flow. Temperatures of the cylinders are fixed, but can be different and, therefore, in the absence of the energy generation the problem will be reduced to the well-known one [3–5].

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Nomenclature

D	diameter of inner cylinder
g	acceleration of gravity
L	gap width of the annulus, $R_o - R_i$
Nu_i, Nu_o	local Nusselt numbers at the inner and outer cylinders, respectively
Pr	Prandtl number, v/χ
R_o, R_i	radii of the outer and inner cylinders, respectively
Ra	Rayleigh number based on the gap width, $\rho g \beta L^3(T_i - T_o)/\mu\alpha$
Ra_T	modified Rayleigh number based on the gap width, $\rho g \beta L^3 T_o q/\mu\alpha$
r	dimensionless radial coordinate
T_i, T_o	temperatures at the inner and outer cylinders, respectively
t	dimensionless time
U, V	velocity components in the radial and angular directions, respectively
u, v	dimensionless velocity components in the radial and angular directions, respectively
F_R, F_θ	the components of a gravity in the radial and angular directions, respectively, referred to the unit volume
c	thermal capacity referred to a mass unit
P	pressure
Q	internal heat generation
q	dimensionless internal heat generation

Greek symbols

α	coefficient of thermal expansion
φ	dimensionless temperature
χ	thermal diffusivity
v	kinematic viscosity
ρ	mean density
σ	ratio of the inner cylinder diameter to gap width, D_i/L
θ	angular coordinate
ψ	dimensionless streamfunction
ω	dimensionless vorticity
μ	dynamic viscosity
β	thermal volumetric expansion coefficient

Convection in systems with heat generation has been studied for a flat layer with uniform source [6] and taking into account energy pumping into vibrational degrees of freedom [7]. Problems associated with the powerful energy generation in nuclear power engineering have been considered as well [8]. However, the problem considered in this paper has features related to the geometry of the system (by contrast to a flat layer, the mode without movement is impossible here) and to capacities (by contrast to a problem of huge capacities of energy generation, in the problem under consideration the temperature mode on the walls can be adjusted as Ra and Ra_T numbers are independent). Moreover, the geometry of coaxial cylinders has some specific features, and the optimization parameter is the temperature field, and not a Nusselt number (its integrated value in the present case is fixed).

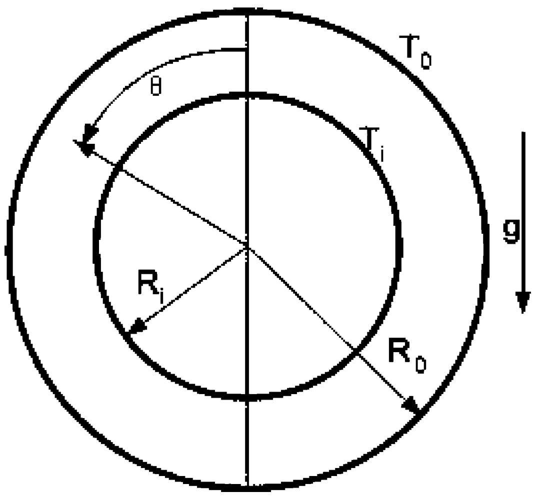

Fig. 1. Geometrical model of the coaxial laser. R_o—radius of the outer cylinder, R_i—radius of the internal cylinder, T_o—temperature on a wall of outer cylinder, T_i—temperature on a wall of the internal cylinder.

2. Formulation of the problem

The configuration to be studied and the coordinate system are shown in Fig. 1. The fluid is contained between two coaxial cylinders of radii R_i and R_o, which are held at temperatures T_o and T_i. The uniform heat generation Q does not depend on coordinates. Density change in the fluid is neglected everywhere except in the buoyancy, and all other physical properties of the fluid are assumed to be constant (Boussinesq approximation).

We consider a two-dimensional problem, and use the cylindrical coordinates (R, θ), where the angular coordinate θ is measured counter-clockwise with respect to the upward vertical axis which contains the centre of the cylinders (Fig. 1).

The dimensional governing equations are

$$\frac{\partial U}{\partial R}+\frac{U}{R}+\frac{1}{R}\frac{\partial V}{\partial \theta}=0, \tag{1}$$

$$\rho\left[\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial R}+\frac{V}{R}\frac{\partial U}{\partial \theta}+\frac{V^2}{R}\right] = -\frac{\partial P}{\partial R}+\mu\left[\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}+\frac{1}{R^2}\frac{\partial^2 U}{\partial \theta^2} -\frac{U}{R^2}-\frac{2}{R^2}\frac{\partial V}{\partial \theta}\right]+F_{\mathrm{R}}, \tag{2}$$

$$\rho\left[\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial R}+\frac{V}{R}\frac{\partial V}{\partial \theta}+\frac{VU}{R}\right] = -\frac{1}{R}\frac{\partial P}{\partial \theta}+\mu\left[\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}+\frac{1}{R^2}\frac{\partial^2 V}{\partial \theta^2} -\frac{V}{R^2}+\frac{2}{R^2}\frac{\partial U}{\partial \theta}\right]+F_{\theta}, \tag{3}$$

$$\rho c\left[U\frac{\partial T}{\partial R}+\frac{V}{R}\frac{\partial T}{\partial \theta}+\frac{\partial T}{\partial t}\right] = k\left[\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}+\frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2}\right]+Q, \tag{4}$$

where U and V are the radial and angular velocities, respectively, P is the pressure, T is the temperature, F_{R} and F_{θ} are the components of a gravity referred to unit of volume, ρ is a density, μ is the dynamic viscosity, c is the thermal capacity referred to a mass unit, k is the thermal conductivity.

It is necessary to note that the possibility of application of Boussinesq approximation has been theoretically examined in general case [9,10]. A comparative numerical analysis for cavity for two-dimensional case has also been carried out [11]. The sense of these results comes to low rate of movement and in the approximation of low rate restrict the considering Rayleigh numbers. But the calculations shows that the Rayleigh numbers at which Boussinesq approximation does not work any more, have a great value. Numerical estimations show that it is necessary to use a full system of hydrodynamic equations in case of $Ra_{\mathrm{T}} \geqslant 3\times 10^8$ [11].

The temperature constituent of the body-force terms can be written as functions of the temperature difference:

$$F_{\mathrm{R}} = g\rho\beta(T-T_{\mathrm{o}})\cos\theta, \tag{5}$$

$$F_{\theta} = g\rho\beta(T-T_{\mathrm{o}})\sin\theta, \tag{6}$$

where T is the temperature of a fluid in a cavity between cylinders and β is the thermal volume expansion coefficient.

The stream function Ψ can be introduced which satisfies the continuity equation by setting:

$$U = R^{-1}\partial\Psi/\partial\theta, \quad V=-\partial\Psi/\partial R. \tag{7}$$

The dimensionless parameters are

$$\psi=\frac{\Psi}{\alpha}, \quad r=\frac{R}{L}, \quad \varphi=\frac{T-T_{\mathrm{o}}}{T_{\mathrm{o}}q}, \quad u=\frac{UL}{\alpha}, \quad v=\frac{VL}{\alpha}, \quad q=\frac{QL^2}{kT_{\mathrm{o}}}, \tag{8}$$

where $\alpha = k/\rho c$ is the thermal diffusivity and $L = R_{\mathrm{o}} - R_{\mathrm{i}}$.

The resulting equations can be simplified by introducing the vorticity ω, defined as

$$\omega = -\nabla^2\psi, \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

—Laplacian in cylindrical coordinates. (10)

The initial system (1)–(4) is reduced to dimensionless governing system:

$$\nabla^2\psi = -\omega, \tag{11}$$

$$\nabla^2\omega = \frac{1}{Pr}\left[\frac{\partial\omega}{\partial t}+u\frac{\partial\omega}{\partial r}+\frac{v}{r}\frac{\partial\omega}{\partial\theta}\right] + Ra_{\mathrm{T}}\left[\sin\theta\frac{\partial\varphi}{\partial r}+\frac{1}{r}\cos\theta\frac{\partial\varphi}{\partial\theta}\right], \tag{12}$$

$$\nabla^2\varphi = u\frac{\partial\varphi}{\partial r}+\frac{v}{r}\frac{\partial\varphi}{\partial\theta}-1+\frac{\partial\varphi}{\partial t}, \tag{13}$$

where $Pr=\mu c/k$ is the Prandtl number and $Ra_{\mathrm{T}} = \rho g\beta L^3 T_{\mathrm{o}} q/\mu\alpha$ is the modified Rayleigh number.

Temperature and velocity distribution into the annulus between coaxial cylinders are determined by the Prandtl number, Rayleigh number, boundary conditions and geometrical parameters of the system.

In the present problem the boundary conditions correspond to two impermeable isothermal walls of cylinders with constant radii and one vertical symmetry axis at $\theta = 0°$ and 180°. The stream function is equal to zero on all boundaries along both walls as well as along the symmetry axis as there are no fluxes through the walls and through the plane. The angular derivative temperatures and vorticity on a line of symmetry disappear.

$$\omega = -\partial^2\psi/\partial r^2. \tag{14}$$

The boundary conditions on the symmetry plane become

$$\psi=\omega=\partial\varphi/\partial\theta=0 \tag{15}$$

while on the inner and outer cylinders they can be written as

$$\psi=u=v=0, \quad \omega=-\partial^2\psi/\partial r^2, \quad \varphi|_{r=r_{\mathrm{i}}} = Ra/Ra_{\mathrm{T}}, \quad \varphi|_{r=r_{\mathrm{o}}}=0, \tag{16}$$

where $Ra = \rho g\beta L^3(T_{\mathrm{i}}-T_{\mathrm{o}})/\mu\alpha$.

We have obtained $q\approx 10^2$ and $Ra_{\mathrm{T}}\approx 2\times 10^4$ for standard parameters of discharge $Q\sim 0.5$ W/sm^3,

$\alpha \sim 3\ \mathrm{sm}^2/\mathrm{s}$, $L \sim 2$ sm, $\mu \approx 2\ \mathrm{sm}^2/\mathrm{s}$ [12]. It is important to notice that the modified Rayleigh number depends on typical size of system as L^5 and that is why Ra_T can vary in sufficiently wide range. However, as one can see from the estimation presented above, the parameter Ra_T in the discharge is sizable and so the convection plays a considerable role.

It is possible to use the standard normalization with the parameter $T_\mathrm{i} - T_\mathrm{o}$ [3–5]. In this case in Eq. (12) the parameter Ra_T will be replaced by Ra. In the right hand side of Eq. (13) the unity will be replaced by Ra_T/Ra, and in Eq. (16) for φ one obtains the standard result: $\varphi|_{r=r_\mathrm{i}} = 1$, $\varphi|_{r=r_\mathrm{o}} = 0$. At both ways of settings results completely coincide. The only exception corresponds to the points where $Ra = 0$ and $Ra_\mathrm{T} = 0$ because the ratio Ra/Ra_T is not defined and application appropriate of settings is impossible.

In the absence of convection Eq. (13) can be written as

$$\nabla^2\varphi = -1, \tag{17}$$

where the boundary conditions are given by Eq. (16). Eq. (17) has the following solution:

$$\varphi = -\frac{r^2}{4} + \frac{r_\mathrm{o}^2 \ln\frac{r}{r_\mathrm{i}} - r_\mathrm{i}^2 \ln\frac{r}{r_\mathrm{o}} - 4\frac{Ra}{Ra_\mathrm{T}} \ln\frac{r}{r_\mathrm{o}}}{4\ln\frac{r_\mathrm{o}}{r_\mathrm{i}}}, \tag{18}$$

where $\sigma = \frac{2r_\mathrm{i}}{r_\mathrm{o}-r_\mathrm{i}}$; $r_\mathrm{i} = \frac{\sigma}{2}$; $r_\mathrm{o} = 1 + \frac{\sigma}{2}$.

The value of $r = r_\mathrm{max}$ which corresponds to the maximum temperature can be determined from the equation $\partial\varphi/\partial r = 0$. We obtain

$$r_\mathrm{max}^2 = \frac{r_\mathrm{o}^2 - r_\mathrm{i}^2 - 4\frac{Ra}{Ra_\mathrm{T}}}{2\ln\frac{r_\mathrm{o}}{r_\mathrm{i}}} = \frac{(1+\sigma) - 4\frac{Ra}{Ra_\mathrm{T}}}{2\ln\frac{2+\sigma}{\sigma}}. \tag{19}$$

It should be noted that if $r_\mathrm{i} < r_\mathrm{max} < r_\mathrm{o}$, the profile of the temperature is nonmonotonic.

The transition to a monotonous profile corresponds to the values $r_\mathrm{max} = r_\mathrm{i}$ and $r_\mathrm{max} = r_\mathrm{o}$. At $r_\mathrm{max} = r_\mathrm{i}$ one obtains from (19)

$$\frac{Ra}{Ra_\mathrm{T}} = \frac{(1+\sigma) - \frac{\sigma^2}{2}\ln\frac{2+\sigma}{\sigma^2}}{4}. \tag{20}$$

At $r_\mathrm{max} = r_\mathrm{o}$

$$\frac{Ra}{Ra_\mathrm{T}} = \frac{(1+\sigma) - 2\left(1+\frac{\sigma}{2}\right)^2\ln\frac{2+\sigma}{\sigma^2}}{4}. \tag{21}$$

The analysis of (20) and (21) shows, that at the absence of convection the nonmonotonic structure arises only for Ra/Ra_T in a range from -0.5 up to 0.5 and faint depends on σ.

Now, setting some initial distributions of temperature and the stream function, using Eqs. (11)–(13) and boundary conditions (15) and (16) it is possible to follow the evolution of this initial distribution, and, in particular, to obtain an expression for a limiting stationary mode if it exists. In the numerical solution of our problem we employ the method of final differences. Finite-difference scheme is standard and the Poisson equation (11) is solved by the method of variable directions, with the chase method used for every direction.

3. Results and discussion

The surface Ra_T (Ra, σ), which separates the region with a two-dimensional convection from the one with the three-dimensional modes is shown in Fig. 2. This surface represents a generalization of the existing results for the dependence $Ra(\sigma)$ in the absence of heat generation ($Ra_\mathrm{T} = 0$). Taking into account that the temperature difference on the inner and outer cylinders can vary both ways, negative values of Ra corresponding to the case $T_\mathrm{i} < T_\mathrm{o}$ have also been taken into consideration.

The critical surface becomes asymmetrical with a change of sign of Ra and with a reduction of σ. This behavior is related to an increase of the difference between the areas of the surfaces of the two cylinders with the decreasing σ.

In Fig. 3 the dependence Ra_T (Ra) for the fixed value $\sigma = 2$ is plotted.

Four areas have been distinguished. I and II areas correspond to a two-dimensional convection, while the III and IV areas correspond to a three-dimensional convection. In the first area (I) convection is accompanied by the formation of two eddies and the temperature gradient varies. However, inside each eddy the gradient does not change sign, while in the second area (II) there is only one eddy and the temperature gradient has a constant sign.

For comparison the dotted lines separating a range of Ra/Ra_T values between -0.63 and 0.49 are shown in Fig. 3. In this case, in the absence of convection the

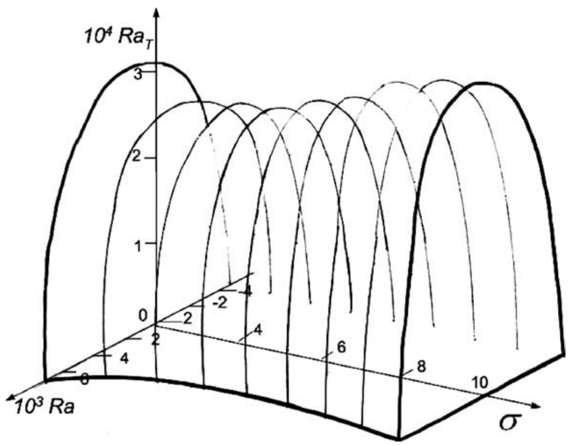

Fig. 2. Critical surface Ra_T (Ra, σ).

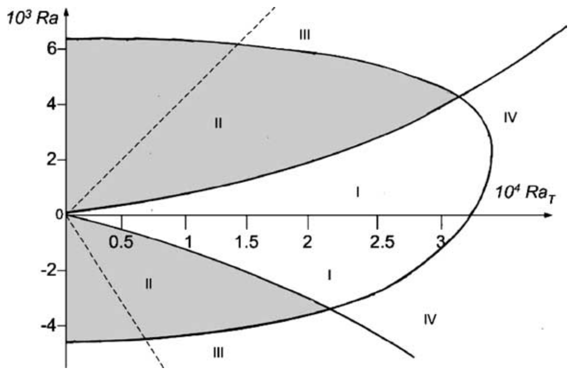

Fig. 3. The dependence Ra (Ra_T) for $\sigma = 2$.

nonmonotonic temperature profile is observed. These values are calculated using Eqs. (20) and (21). One can readily see that the convection reconstructs the temperature distribution to a very large extent al though the existence of an inhomogeneous distribution at small values of the parameter Ra/Ra_T is preserved.

A typical picture of the isotherms and streamlines for the area I is presented in Fig. 4a, and for the second area it is presented in Fig. 4b.

There is only one eddy in the absence of the heat generation in a two-dimensional mode according to the direction of the temperature gradient. The temperature gradient has a unique sign inside this eddy. The same behavior is found in another problem where heat generation is taken into account although the presence of convection results in a number of changes in the inhomogeneous temperature distribution.

One notes that the eddies are disintegrated in the third (III) and the fourth (IV) areas. This corresponds to a transition into a new regime which has not been studied in detail in this paper. The corresponding solution, however, is well-known for the case of $Ra_T = 0$. One finds different two- and three-dimensional structures in this case.

Now let us consider the same system for the prospects of laser construction in the case of equal temperatures at both boundaries ($Ra = 0$).

We first compare results without convection. The equation $\nabla^2\varphi = -1$ can readily be solved with the boundary conditions $\varphi|_{r_i,r_o} = 0$:

$$\varphi = -\frac{r^2}{4} + \frac{r_o^2 \ln\frac{r}{r_i} - r_i^2 \ln\frac{r}{r_o}}{4\ln\frac{r_o}{r_i}}. \tag{22}$$

In the limit $\sigma \to 0$ one obtains the value 0.25 at the maximum which corresponds to a solution of a similar problem with heat generation for a cylinder where $\varphi = (r_o^2 - r^2)/4$. By contrast to the cylindrical geometry, the coaxial geometry enables one to reduce the maximal temperature in the system by a factor of two. The maximal temperature steeply decreases with increasing σ and only marginally differs from the limiting value 0.125 for $\sigma \geqslant 1$. We consider now the same problem taking convection into account.

Streamlines and isotherms for $\sigma = 2$, $r_i = 1$, $r_o = 2$, $Ra_T = 3 \times 10^4$ are shown in Fig. 5.

One can readily see from the picture, there the angular dependence of the temperature is strongly inhomogeneous. The maximal value of the temperature exceeds the appropriate maximum temperature for the case without convective. In Fig. 6 one finds the dependence of the maximum possible temperature in the system on the modified Rayleigh number at different values of σ. The maximal temperature value grows with the increasing Ra_T for the coaxial cylinders system.

The reason for this is the existence of the two eddies. Indeed, if one considers the top area of the system (Fig. 7) and compare the heat flow with and without convection, one can readily see that in the case of con-

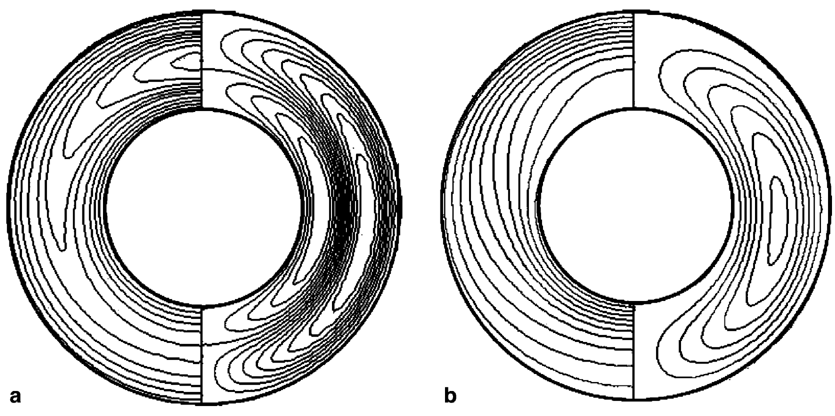

Fig. 4. Isotherms and streamlines for $\sigma = 2$: (a) $Ra = 1 \times 10^2$, $Ra_T = 3 \times 10^4$ (area I on Fig. 3); (b) $Ra = 2 \times 10^3$, $Ra_T = 5 \times 10^3$ (area II on Fig. 3).

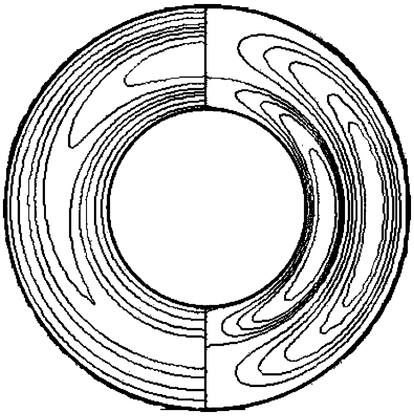

Fig. 5. Isotherms and streamlines for $\sigma = 2$, $Ra_T = 3 \times 10^4$.

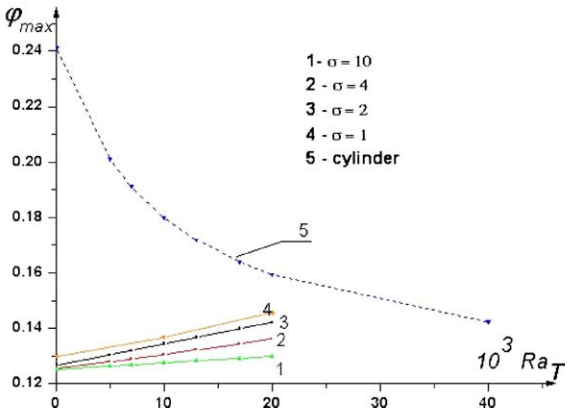

Fig. 6. The maximal temperature as a function of Ra_T for different values of σ.

vection there exists the additional influx of energy into the system from the side areas because incoming streams are heated up more than outgoing ones. This also leads to an increase of the maximal temperature. The temperature distribution as a function of angle can be characterized by the angular heterogeneity of the thermal flow at the wall. If one splits the surface into two parts, for the $\sigma = 2$ at $Ra_T = 3.5 \times 10^4$, that corresponds to Fig. 5, Nusselt numbers are

$$\int_0^{\pi/2} r\frac{\partial\varphi}{\partial r}\bigg|_{r=r_o} = 1.52, \quad \int_{\pi/2}^{\pi} r\frac{\partial\varphi}{\partial r}\bigg|_{r=r_o} = 1.29,$$

$$\int_0^{\pi/2} r\frac{\partial\varphi}{\partial r}\bigg|_{r=r_i} = 0.92, \quad \int_{\pi/2}^{\pi} r\frac{\partial\varphi}{\partial r}\bigg|_{r=r_i} = 0.86.$$

Certainly, the total flux is preserved. It depends only on σ and is equal to $\pi(1+\sigma)/2$ for the semi-circumference.

The problem which accounts for the volumetric heat generation in the cylinder is solved in a similar way using the Cartesian coordinates. In Fig. 8 one finds typical isotherms and streamlines while the dependence of the maximal temperature on Ra_T is shown in Fig. 6. For the cylinder geometry the temperature decreases with the increasing heat generation and at some value of Ra_T it becomes even smaller than in the case of two coaxial cylinders.

However, it is necessary to recognize, that it is impossible to carry out the direct comparison of the results for the two geometries because the parameter Ra_T is scaled with the radius of the cylinder for the cylindrical geometry while for the coaxial system the parameter is scaled with the gap width. Therefore, for the same value of Ra_T the cross section area will be different.

If one considers the system with the same heat generation and the same cross section, the appropriate value of Ra_T in the system of coaxial cylinders will be much

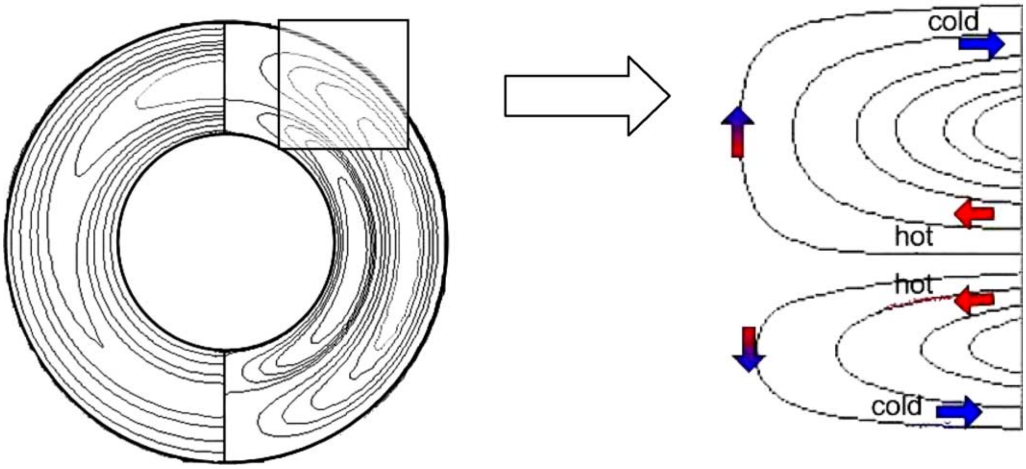

Fig. 7. Convective flows at the top point of the coaxial cylinder system.

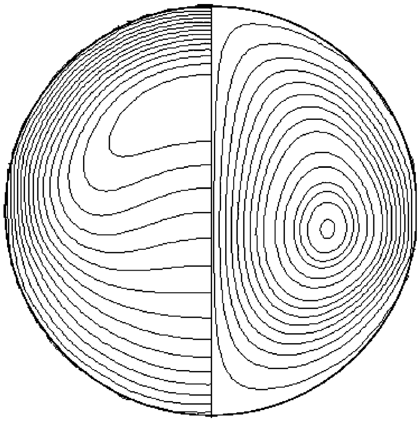

Fig. 8. Isotherms and streamlines for the cylinder when $Ra_T = 2 \times 10^4$.

smaller (namely, be a factor of $(1 + \sigma)^{5/2}$). However, the smaller value of Ra_T means the essentially smaller speed of convection. It should be noted that for the same value of Ra_T the speed of convection is much smaller in the system of coaxial cylinders because of the presence of the two eddies. The convection in the two-dimensional case results in completely different properties of the systems with one cylinder and two coaxial cylinders, respectively. In the former system the maximal temperature is strongly reduced while in the coaxial cylinders system it increases slightly. By an order of magnitude these changes are comparable to a difference of the maximal temperatures in the absence of convection.

In the present work, we did not consider the limiting transition to the single cylinder by reducing the radius of the small cylinder in the coaxial system. Such limiting geometry is not very interesting from the practical point of view, since it is impossible to keep the temperature constant for a thin cylinder. From the theoretical point of view this transition is not straightforward because of the existence of a return eddy which covers more area than that of the internal cylinder. Even in the case of $\sigma = 0$ the system differs from the cylinder due to the existence of the central point with the fixed temperature even though the heat flow to the centre in this case approaches zero.

It should be noted that calculations presented in this paper correspond to a two-dimensional model and to the case of moderately high Rayleigh numbers. Moreover, heat generation is taken into account in this model in a relatively simple way. However, even such a simple model enables one to reveal the basic features of the system. The structure of the convective currents changes with the increasing of Ra_T, and therefore the analysis of the convective flows should play an extremely important role in the optimization of the geometry of a laser system.

4. Conclusions

1. We have considered the problem of convection in a system of two horizontal coaxial cylinders with internal heat generation and different temperatures at the boundaries. The mathematical model which describes the two-dimensional convection has been proposed and the corresponding hydrodynamic parameters have been calculated.
2. The critical surface Ra_T (Ra, σ) which corresponds to a generation of the known results for $Ra_T = 0$ has been described.
3. It has been shown that depending on the parameters of a problem there exist two different distributions of two-dimensional currents—with one and two vortices.
4. The convection in the horizontal cylinder with constant heat generation has been investigated and the values of the maximum temperature in the system have been compared for the two systems with different geometry. It is shown, that in the case of convection in the cylinder the maximal temperature decreases with the increasing heat generation, while in the system of coaxial cylinders the maximum temperature increases if the radius of the internal cylinder is not too small.

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